# I. Power Number

## 1. Definition

How does power consumption relate to operating variables? The key dimensionless group is the Power Number (Po).

$$Po = \frac{\text{External Force Exerted}}{\text{Inertial Force Imparted}} = \frac{\text{Power}}{\rho N^3 d^5}$$

Power = External Power to Agitator
$\rho$ = Liquid Density
N = Impeller Rotational Speed (rpm)
d = Impeller Diameter

Other important dimensionless groups:

$$Re = \frac{\text{Inertial Forces}}{\text{Viscous Forces}} = \frac{\rho d^2 N}{\mu}$$

Osborne Reynolds
(no apostrophe)

$$Fr = \frac{\text{Inertial Forces}}{\text{Gravitational Forces}} = \frac{N^2 d}{g}$$

William “Frew-d”

g = gravitational acceleration

## 2. Correlation with other Dimensionless Groups

In general, the Power Number for *ungassed* systems is related to the Reynolds Number and the Froude Number:

$$Po = c\,(Re)^x\,(Fr)^y$$

In a well-baffled vessel, the gravitational forces are minimal, and the Power Number for ungassed systems is related only to the Reynolds Number:

$$Po = c\,(Re)^x$$

Bioreactors are typically well-baffled.

a) For laminar flow (Re<10), Po decreases linearly with the logarithm of Re. Thus, x = -1 and the power absorbed is a function of the fluid viscosity:

$$Po = c\,(Re)^{-1}$$

or,

$$\frac{Power}{\rho N^3 d^5} = c\left(\frac{\mu}{\rho d^2 N}\right)$$

$$Power = c d^3 N^2 \mu$$

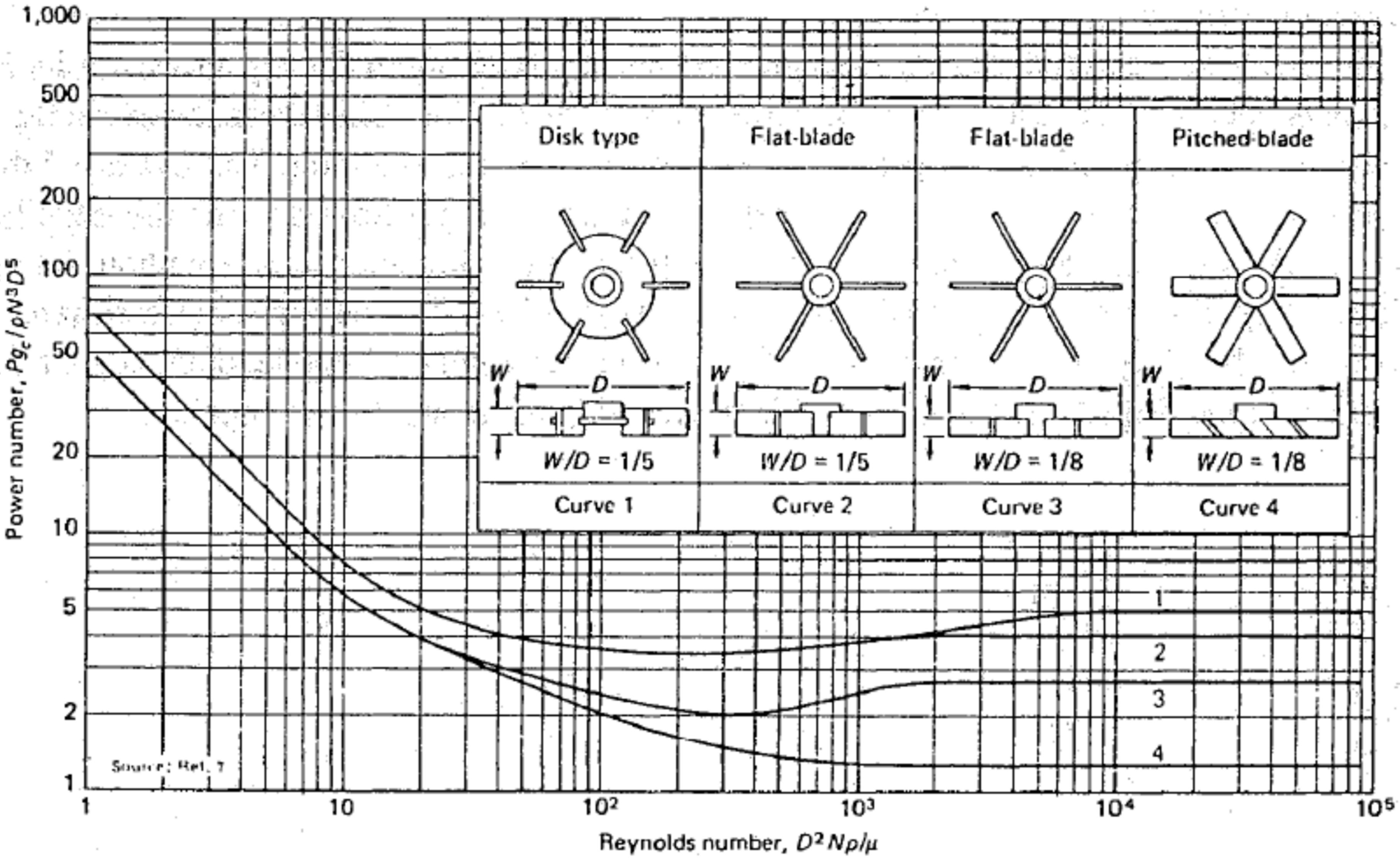
b) For transition flow (10<Re<10000), Po is a complex function of Re.

c) For turbulent flow (10000<Re), Po is a constant (independent of Re). Thus, x = 0, and the power absorbed is not a function of the fluid viscosity.

$$Po = c$$

$$\frac{Power}{\rho N^3 d^5} = c$$

$$Power = c\rho N^3 d^5$$



Disk type
Flat-blade
Flat-blade
Pitched-blade
W
D
W/D = 1/5
W/D = 1/5
W/D = 1/8
W/D = 1/8
Curve 1
Curve 2
Curve 3
Curve 4
1
2
3
4
Power number, $Pg_c/\rho N^3D^5$
1,000
500
200
100
50
20
10
5
2
1
Reynolds number, $D^2N\rho/\mu$
1
10
$10^2$
$10^3$
$10^4$
$10^5$
Source: Ref. 7

Notes:

Axial-flow impellers usually have lower values of Po compared to radial-flow impellers.

These relationships are for ungassed systems (i.e., no aeration). Aeration has the effect of lowering the liquid density and liquid viscosity compared to the ungassed liquid.

For gassed systems, must use correlations, such as:

$$\frac{P_{gassed}}{P_{ungassed}} = 0.10 \left( \frac{Q}{NV_L} \right)^{-0.25} \left( \frac{N^2 d^4}{gBV_L^{2/3}} \right)^{-0.20}$$

Q = Gas volumetric flowrate
B = Impeller blade width

(Hughmark, 1980)

Several correlations are available to predict ungassed power consumption (Oyamma and Endoh, 1955; Michel and Miller, 1962).

Example:

A 10,000 L vessel has a flat-blade impeller (W/D=0.2, B = 0.5 cm) having a diameter of 1.0 m, and the impeller will operate at 100 rpm.

1) What power is required to mix ungassed pure water?
2) What power is necessary if the water is gassed at a volumetric flowrate of 4000 L/min?

Solution:

1) What power is required to mix ungassed pure water?

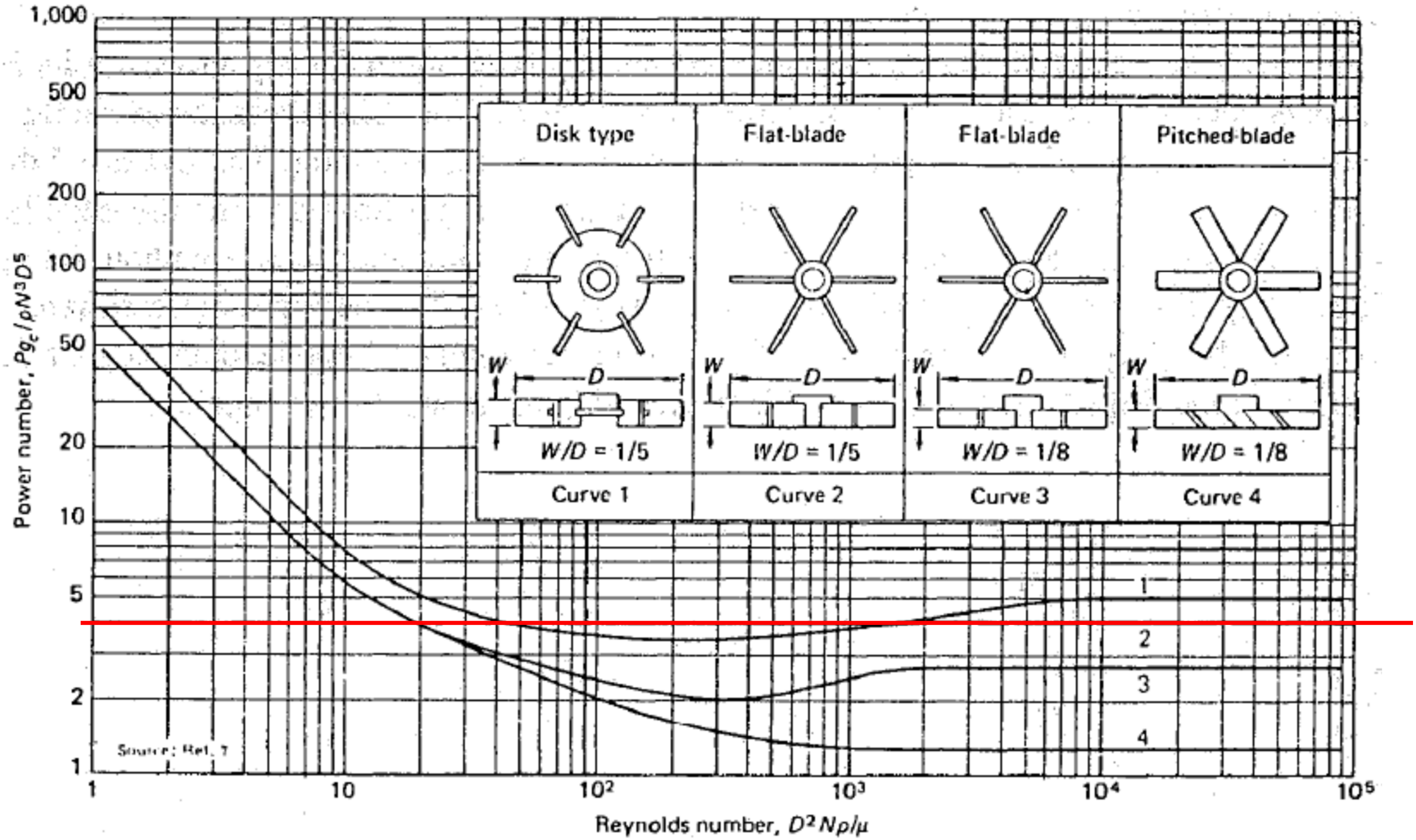
$\mu = 1 \text{ cP} = 0.001 \text{ Pa·s} = 0.001 \text{ N·s/m}^2 = 0.001 \text{ kg/s·m}$

$\rho = 1 \text{ g/mL} = 1000 \text{ kg/m}^3$

$N = 100 \text{ min}^{-1}\text{·}(1 \text{ min}/60 \text{ s}) = 1.67 \text{ s}^{-1}$

$$Re = \frac{\rho d^2 N}{\mu} = \frac{(1000 \text{ kg/m}^3)(1.0 \text{ m})^2(1.67 \text{ s}^{-1})}{(0.001 \text{ kg/s·m})}$$

Re = 1,670,000 … definitely turbulent



1,000
500
200
100
50
20
10
5
2
1
Power number, $Pg_c/\rho N^3D^5$
Disk type
Flat-blade
Flat-blade
Pitched-blade
W
D
W/D = 1/5
W/D = 1/5
W/D = 1/8
W/D = 1/8
Curve 1
Curve 2
Curve 3
Curve 4
1
2
3
4
Source: Ref. 7
1
10
$10^2$
$10^3$
$10^4$
$10^5$
Reynolds number, $D^2N\rho/\mu$

## Solution (cont'd):

From plot Po = c = 4

So,

Power = $c\rho N^3 d^5$

Power = $(4)(1000\ kg/m^3)(1.67\ s^{-1})^3(1.0\ m)^5$

Power = 18520 W

Power = 18520 W (1 hp/745.7 W) = 24.8 hp

Solution (cont'd):

2) What power is necessary if the water is gassed at a volumetric flowrate of 4000 L/min?

$$\frac{\text{Power}_{gassed}}{\text{Power}_{ungassed}} = 0.10\left(\frac{Q}{NV_L}\right)^{-0.25}\left(\frac{N^2d^4}{gBV_L^{2/3}}\right)^{-0.20}$$

$$\frac{Q}{NV_L} = \frac{(4000\ \text{L/min})(\text{min}/60\ \text{s})}{(1.67\ \text{s}^{-1})(10000\ \text{L})} = 0.0010$$

$$\frac{N^2d^4}{gBV_L^{2/3}} = \frac{(1.67\ \text{s}^{-1})^2(1.0\ \text{m})^4}{(9.81\ \text{m/s}^2)(0.005\ \text{m})(10\ \text{m}^3)^{2/3}} = 12.2$$

Solution (cont'd):

2) What power is necessary if the water is gassed at a volumetric flowrate of 4000 L/min?

$$\frac{\text{Power}_{gassed}}{\text{Power}_{ungassed}} = 0.10\ (0.0010)^{-0.25}(12.2)^{-0.20} = 0.341$$

$$\text{Power}_{gassed} = 0.341 \times 18520\ \text{W} = 6315\ \text{W}$$

$$= \underline{\underline{8.5\ \text{hp}}}$$

# J. Scale-up and Correlations Involving $k_L a$

## 1. Introduction

The typical scenario is to conduct early development experiments at a small scale (1 L), and then scale the process up to production scale (2,000 – 1,000,000 L). With a lot of luck, the microbes will behave the same at the larger scale as they do at the smaller scale (e.g., generate the product). Unfortunately, scale-up of a biological process is extremely difficult because of the presence of so many variables, and not all of the variables can be maintained during scale-up. Experience is the strongest guide.

Parameters used in scale up can be categorized:

1) Physical properties ($\mu$, $\sigma$, $\rho$, $D_L$, $c^*_{O2}$)
2) Biochemical properties ($\mu_{MAX}$, $q_O$)
3) Operational conditions ($U_{SF}$, N, Power/$V_L$)
4) Geometric parameters (type of bioreactor, d)

Typically cannot change (or don't want to change) physical properties or biochemical properties.

Successful scale-up benefits from:

1) Scaling-up gradually (scale up volume ratio of 10:1 or less) from laboratory scale to pilot scale and pilot scale to production scale (e.g., 1 L → 10 L → 100 L → 1,000 L → 10,000 L). Piloting a process gives the engineer the chance to learn about ‘scale-up issues’.
2) Using correlations to predict how parameters like $k_La$ scale.
3) Experience.

## 2. Correlations for $k_L a$

There are generally two classes of correlations, those that involve dimensionless groups and those that do not. Empirical correlations always should be considered in the context of conditions for which the data were collected. The correlations may not be transferable to other situations. In particular, different correlations exist between the different type of reactors:

a) Stirred tank bioreactor (only ones covered here)
b) Airlift bioreactor
c) Bubble column bioreactor

## a. Correlations not using Dimensionless Groups

A typical correlation will involve a combination of power/volume ratio (Power/$V_L$), superficial gas velocity ($U_{SF} = 4Q/\pi D_{REACTOR}^2$), and liquid viscosity ($\mu$). Other variables are sometimes included depending on the wishes of the individual correlating the data.

$$k_L a = C\, f(N)\, (U_{SF})^a\, (Power/V_L)^b\, (\mu)^c$$

where a, b, c are empirical exponents often having ranges:

$0.3 \le a \le 0.7$ $0.4 \le b \le 1.0$ $-0.4 \le c \le -0.7$

## Examples of correlations:

$U_{SUP}$ (m/s); $k_La$ ($s^{-1}$); $V_L$ (L); Power ($W/m^3$)
40% accuracy
Ref: Van't Riet, 1979

Water:

$$k_La = 0.026 \ (U_{SUP})^{0.5} \ (Power/V_L)^{0.4}$$

$500\ W/m^3 < Power/V_L < 10{,}000\ W/m^3$
$2L < V_L < 2{,}600L$

Ionic Solutions:
(0.10 – 0.40 M)

$$k_La = 0.0020 \ (U_{SUP})^{0.2} \ (Power/V_L)^{0.7}$$

$500\ W/m^3 < Power/V_L < 10{,}000\ W/m^3$
$2L < V_L < 4{,}400L$

Interpretations:

Water: $k_L a = 0.026 \ (U_{SUP})^{0.5} \ (Power/V_L)^{0.4}$

Ionic Solutions: $k_L a = 0.0020 \ (U_{SUP})^{0.2} \ (Power/V_L)^{0.7}$

- $k_L a$ depends more on $Power/V_L$ in ionic solutions than in water.
- $k_L a$ is larger in ionic solutions than in water because of reduced bubble size.

Note:

For $U_{SUP} = 0.1$ m/s and $Power/V_L = 1000 \ W/m^3$

$k_L a$ (water) = 0.013 $s^{-1}$

$k_L a$ (ionic) = 0.016 $s^{-1}$

## b. Correlations using Dimensionless Groups

Another type of empirical equation correlates dimensionless groups containing the mass transfer coefficient with dimensionless groups not containing the mass transfer coefficient. Typical dimensionless groups used include:

| | |
|---|---|
| Sherwood (Sh): | $k_L a \cdot d^2/D_L$ |
| Stanton (St): | $k_L a \cdot V_L/Q$ |
| Reynolds (Re): | $\rho \cdot N \cdot d^2/\mu$ |
| Schmidt (Sc): | $\mu/\rho D_L$ |
| Weber (We): | $\rho \cdot N^2 \cdot d^3/\sigma$ |
| Froude (Fr): | $N^2 \cdot d/g$ |

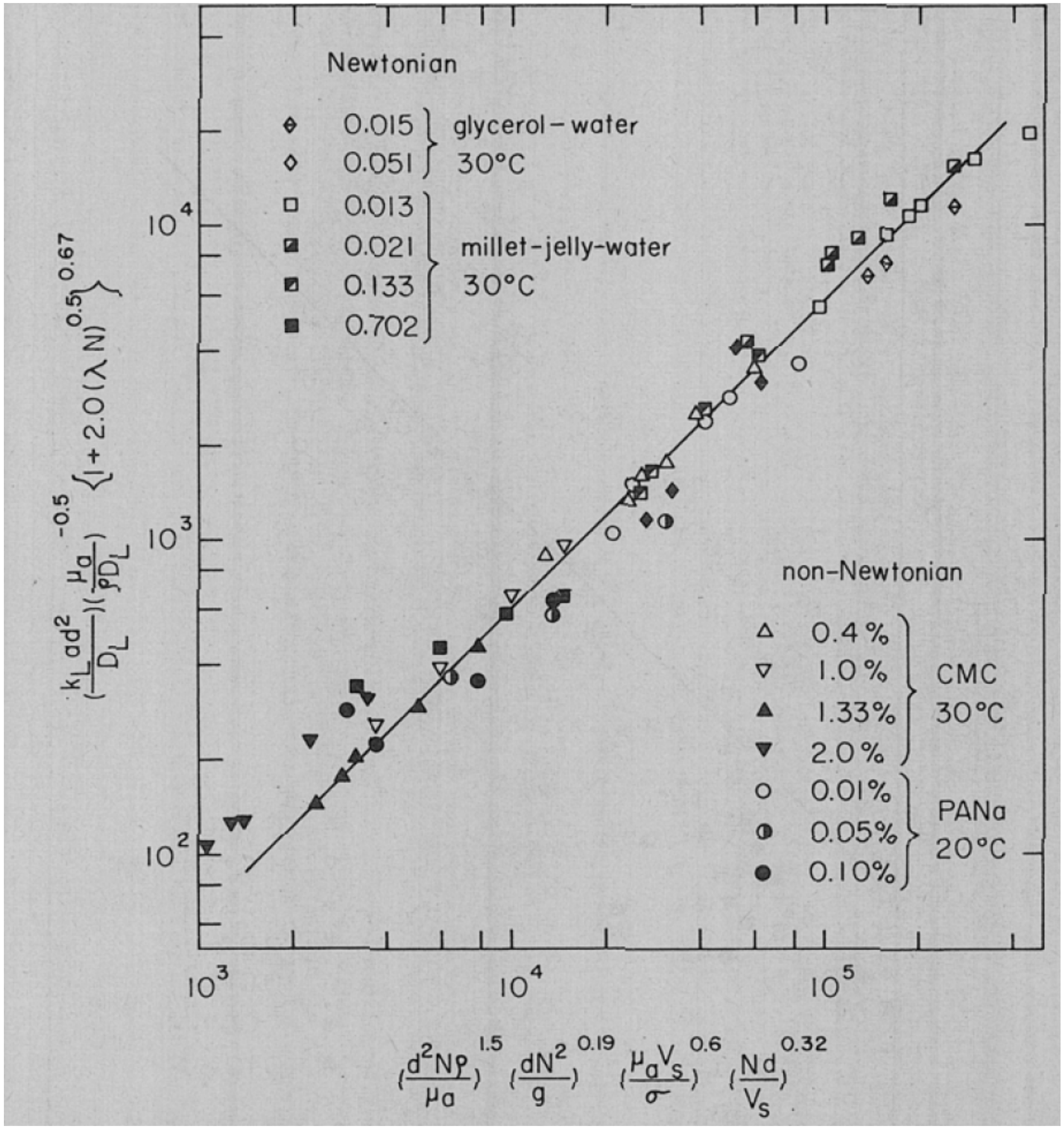
Examples of correlation:

Non-Newtonian and Newtonian Fluids:

$$Sh = 0.060\ Re^{1.5}\ Fr^{0.19}\ Sc^{0.5} \left( \frac{\mu \cdot U_{SF}}{\sigma} \right)^{0.6} \left( \frac{N \cdot d}{U_{SF}} \right)^{0.32}$$

d (diameter of impeller)
$D_L$ (liquid diffusivity)
$\rho$ (liquid density)
N (rotation of impeller, $s^{-1}$)
$\mu$ (apparent viscosity, g/cm·s)
$\sigma$ (surface tension, $g/s^2$)

Ref: Yagi and Yoshida, 1975



Newtonian
0.015 glycerol-water
0.051 30°C
0.013
0.021 millet-jelly-water
0.133 30°C
0.702
non-Newtonian
0.4 %
1.0 % CMC
1.33% 30°C
2.0 %
0.01%
0.05% PANa
0.10% 20°C
$10^2$
$10^3$
$10^4$
$10^5$
$(\frac{k_L a d^2}{D_L})(\frac{\mu_a}{\rho D_L})^{-0.5}\{1+2.0(\lambda N)^{0.5}\}^{0.67}$
$(\frac{d^2 N \rho}{\mu_a})^{1.5}(\frac{dN^2}{g})^{0.19}(\frac{\mu_a V_s}{\sigma})^{0.6}(\frac{Nd}{V_s})^{0.32}$

The problem with all the dimensionless groups is that they hide the relationship between individual parameters (which appear in multiple groups). According to correlation by Yagi and Yoshida, 1975:

$k_L a \propto N^{2.20}$

$k_L a \propto \mu^{-0.40}$

$k_L a \propto \rho^{1.00}$

$k_L a \propto d^{1.51}$

$k_L a \propto U_{SUP}^{0.28}$

Conclusion from this analysis is that changing impeller speed has the largest effect on mass transfer coefficient.

## 3. Criteria used for Bioreactor Scale-Up

Scale-up is typically accomplished by one of four criteria:

1) Constant Power/$V_L$
2) Constant $k_L a$
3) Constant OTR
4) Constant dissolved oxygen ($c^l_{O2}$)

It is not possible to maintain ratios of all parameters during scale-up.

# References

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